Power Efficient Dynamic Bandwidth Allocation Algorithm in OFDMA-PONs

Weizhi You, Lilin Yi, Silu Huang, Jian Chen, and Weisheng Hu

***Abstract*—We propose two dynamic bandwidth allocation (DBA) algorithms by considering three dimensions, i.e., time slot, subcarrier, and bit allocation, to minimize the transmitting power of optical network units (ONUs) in orthogonal frequency division multiplexing access passive optical networks (OFDMA-PONs). In the first proposed algorithm, we make use of the three-dimensional DBA algorithm from wireless OFDM, modify it according to the characteristics of the fiber channel, and apply it to upstream access in OFDMA-PONs to reduce the transmitting power of ONUs. In order to further minimize the total transmitting power, we propose the second algorithm by allocating the subcarrier and bit in the same procedure. The complexity of the two proposed algorithms was evaluated by polynomial time. From simulation results, the transmitting power of ONUs under a certain bit error rate is reduced by ~30% and ~50% using the first and the second proposed algorithms, respectively, compared with the traditional two-dimensional DBA algorithm in OFDMA-PONs.**

***Index Terms*—Dynamic bandwidth allocation; Orthogonal frequency multiplexing access passive optical networks; Power efficiency.**

I. Introduction

Recently with the rapid development of network technologies and the exponential growth of digital services, the passive optical network (PON) has required a higher access data rate [1,2]. The PON comprises a single optical line terminal (OLT) residing in the central office and connecting a set of optical network units (ONUs) located at the customers' premises. Due to the advantages of large capacity, efficient and flexible multiple address access, high spectral efficiency, dynamic bandwidth allocation (DBA), etc., orthogonal frequency division multiplexed access passive optical networks (OFDMA-PONs) have become the focus of PON technologies. In this paper, we mainly address two issues in OFDMA-PONs: DBA and power consumption of ONUs. Both of the issues are of great importance in practice. DBA can improve the performance of the system, including system throughput, channel utilization, etc. Meanwhile, minimizing power consumption is always necessary in practical application. A lot of solutions for lowering the power consumption of OFDMA-PONs [3–6] have been proposed. But most of those algorithms are focused on the hardware level and the media access control (MAC) layer [7,8]. Little research has been performed to lower power consumption from the algorithm level in OFDMA-PONs.

Besides, early related works were mainly based on the two-dimensional scheduling algorithm, i.e., time slot and subcarrier allocation [8–11]. Modulation scheme allocation among subcarriers, i.e., bit allocation and the transmitting power optimization problem, is rarely considered. The adaptively modulated optical OFDM (AMO-OFDM) [12] scheme takes advantage of the modulation allocation characteristic of OFDM signals. However, the purpose of AMO-OFDM is to mitigate the effects of the peak-to-average power ratio (PAPR) rather than lower the transmitting power. In wireless OFDM systems, three-dimensional DBA algorithms have been proposed to minimize the transmitting power [13–15]. But due to the different channel characteristics, the algorithms cannot be directly used in optical fiber systems. Besides, those algorithms are not optimized to further reduce power consumption.

In this paper, first we transformed the problem of minimizing the total transmission power of ONUs in OFDMA-PONs with each ONU's traffic requirement satisfied to a mathematical model. Then, based on the fiber channel characteristics, we proposed two optimized three-dimensional DBA algorithms to solve this model for minimizing the total transmission power of ONUs. In the first proposed algorithm, considering the fiber channel characteristics, we modified the wireless OFDM three-dimensional DBA algorithm [15] and applied it to upstream access in OFDMA-PONs. This algorithm divides the problem into two procedures: subcarrier allocation and bit allocation. However, the two procedures are not independent of each other. Subcarrier allocation will influence the result of bit allocation. And similarly, bit allocation will impact the allocation result of subcarriers. As a consequence, the first proposed algorithm is a suboptimal but not optimal algorithm to solve the proposed mathematical model to minimize the transmitting power. In order to further minimize the transmitting power of ONUs, we

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proposed the second algorithm by allocating the subcarrier and bit in the same procedure to solve the proposed mathematical model. From simulation results, compared with the traditional two-dimensional DBA algorithm in OFDMA-PONs, the transmitting power of ONUs under a certain bit error rate (BER) was reduced by ~30% and ~50% using the first and the second proposed algorithms, respectively. The performance of the second algorithm was better than that of the first algorithm. Furthermore, by the complexity analysis, we found that the time complexity of the two proposed algorithms is polynomial.

The paper is organized as follows. In Section II, we introduce the system model. The proposed algorithms are illustrated in Section III. In Section IV, we analyze the time complexity of the algorithms and the simulation results. Section V is the conclusion.

II. System Model

A. System Structure

The structure of the OFDMA-PON system is shown in Fig. 1. There are three major components in the OFDMA-PON system, namely, the OLT, the optical splitter-based optical distribution network (ODN), and the ONU. The OLT broadcasts downstream traffic data flow from the central office to each ONU and transfers upstream traffic data flow from each ONU to the central office. ONUs selectively receive downstream frames broadcasted by OLT and transfer them to users. The ODN collects data from each ONU in the upstream and splits data from the OLT to ONUs in the downstream.

The upstream and downstream data traffic is transmitted over one optical wavelength channel. The channel can be further divided into OFDM subcarriers, which is shown in Fig. 1. Each OFDM subcarrier can be allocated to different ONUs in different time slots.

B. Model of Power Consumption

We assumed that the system has N subcarriers and K ONUs and the kth ONU has a data rate request of R_k bit. We assumed that channel gains on all the subcarriers are known for the OLT, and then different subcarriers with different numbers of bits per OFDM symbol to be transmitted on each subcarrier are assigned to ONUs, i.e., subcarrier allocation and bit allocation according to DBA algorithms, as illustrated in Fig. 2. In Fig. 2(a), the traditional two-dimensional allocation without bit adaptive modulation is given for comparison. Figure 2(b) shows the principle of three-dimensional allocation with time slot, subcarrier, and bit allocation. The number of bits assigned to each OFDM subcarrier corresponds to the modulation scheme. For instance, four bits assigned to a subcarrier means that the modulation format is 16-QAM. Depending on the modulation scheme, the transmitting power of ONUs can be adjusted so that the receiver can demodulate the original data with minimum transmitting power.

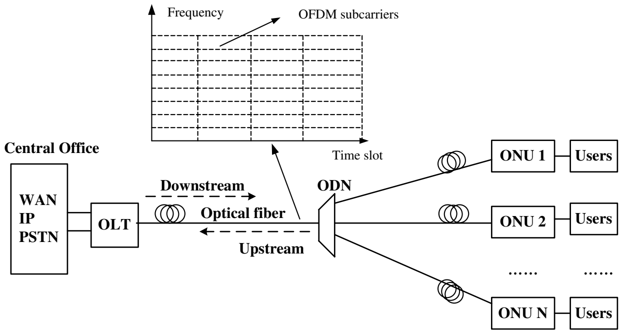
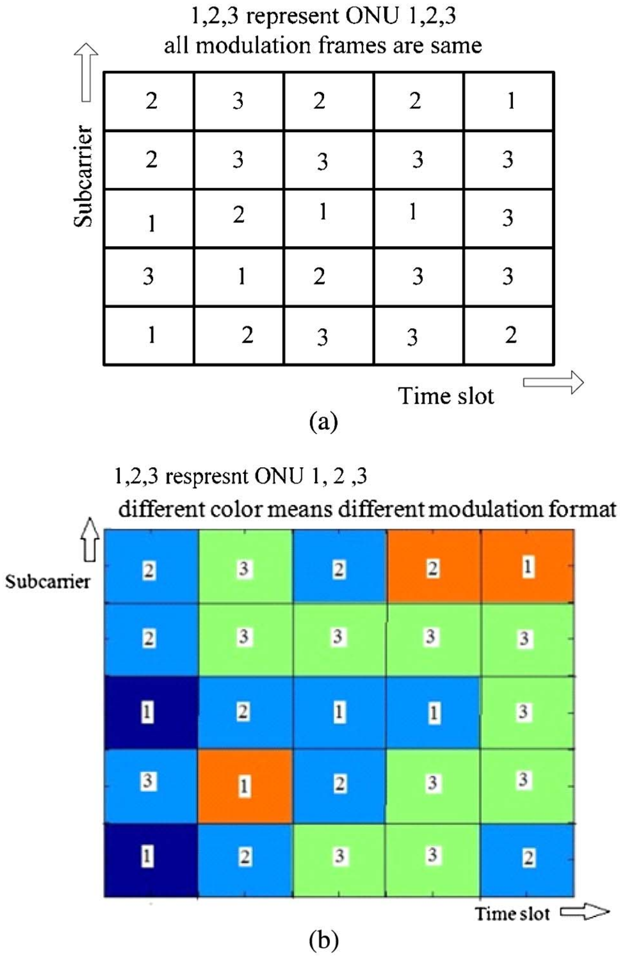


Fig. 1. Structure of OFDMA-PON system.

We defined $c_{k,n}$ as the number of bits assigned to the nth subcarrier of the kth ONU. $c_{k,n}$ takes value in the set $D = \{1, 2, \ldots, M\}$, where M is the maximum number of bits that can be transmitted by each OFDM subcarrier. This illustrates that the modulation format is from 2-QAM to 2^M – QAM. In this system model, there are two main constraints as follows:

1) Each subcarrier can only be used by one ONU within each time slot.



2	3	2	2	1
2	3	3	3	3
1	2	1	1	3
3	1	2	3	3
1	2	3	3	2

2	3	2	2	1
2	3	3	3	3
1	2	1	1	3
3	1	2	3	3
1	2	3	3	2

Fig. 2. (a) Two-dimensional allocation. (b) Three-dimensional allocation.

2) Data rate requirements from all ONUs should be met.

The goal of our algorithm is to find the best assignment of $c_{k,n}$ ($k \in \{1,2,\ldots,K\}$, and $n \in \{1,2,\ldots,N\}$) within each time slot so that the total transmitting power P_T can be minimized when the traffic requirement of each ONU is met. Mathematically, we can formulate the problem as

$$P_T = \min_{c_{k,n} \in D} \sum_{n=1}^{N} \sum_{k=1}^{K} P_{k,n}, \tag{1}$$

where $P_{k,n}$ is

$$P_{k,n} = \frac{f(c_{k,n})}{a_k^2}, \tag{2}$$

where a_k denotes as the channel gain of the kth ONU and $f(c)$ represents the required power for supporting c bits/symbol for a given BER P_e, which can be expressed as [16]

$$f(c) = \frac{N_0}{3}\left[Q^{-1}\left(\frac{P_e}{4}\right)\right]^2 (2^c - 1), \tag{3}$$

where $Q(x) = (1/\sqrt{2\pi}) \int_x^\infty e^{-t^2/2} \mathrm{d}t$, and N_0 is the noise power spectral density.

Equation (1) is subjected to the following two constraints:

C1: For all $n \in \{1,2,\ldots,N\}$, if there exists a k with $c_{k',n} \neq 0$, then $c_{k',n} = 0$, for all $k' = k$.
C2: For all $k \in \{1,2,\ldots,K\}$, $R_k = \sum_{n=1}^{N} c_{k,n}$.

Note that C1 ensures each OFDM subcarrier can only be occupied by one ONU within each time slot and C2 guarantees that the OLT can satisfy the data rate requirement of each ONU.

From the above analysis, we know that the problem of minimizing the total transmission power of ONUs with each ONU traffic requirement met is transformed to the mathematical model Eq. (1) with two constraints, C1 and C2. Therefore, what we should do is find the proper algorithms to solve this model and minimize the total transmitting power of ONUs.

Since large constellation maps, corresponding to a larger c value, lead to increased BER, the signal transmitting power should be higher for larger constellation maps to achieve a fixed BER. The electrical power contributes a large part of the ONU total transmitting power because the optical power of the ONU is only 5–10 mW, but the electrical power is around 6–50 W [17]. Furthermore, no matter whether we use the two-dimensional algorithms or the three-dimensional algorithms, the optical power of ONUs is similar. Therefore the optical power consumption can be ignored when we consider the relative value of transmitting power from all ONUs for algorithm evaluation. Hence, the total transmitting power we considered in this paper is the electrical power consumption rather than the optical one.

III. PROPOSED SCHEDULING ALGORITHMS

Based on the characteristics of the fiber channel of the OFDMA-PON system, we propose two optimized three-dimensional DBA algorithms for upstream access in OFDMA-PONs to solve the proposed mathematical model and find the best assignment of $c_{k,n}$ within each time slot. The total transmitting power P_T can be minimized when the traffic requirement of each ONU is met. In this section, we first define some symbols used in the algorithms. Then the two proposed algorithms are introduced respectively. Finally, the minimized transmission power P_T is expressed.

A. *Explanations of Symbols*

Let S_k represent the number of OFDM subcarriers occupied by the kth ONU. Set L_k record the OFDM subcarriers occupied by the kth ONU. Obviously, $L_i \cap L_j = \varnothing$ ($\forall\, i \neq j$). $D_{\min}$ and $D_{\max}$ are the minimum and maximum number of bits per OFDM symbol that can be transmitted by each subcarrier. For instance, from $D = \{1,2,\ldots,M\}$, we can know that $D_{\min} = 1$ and $D_{\max} = M$. ΔP_k is the transmission power changing in the kth ONU.

B. *First Proposed Algorithm*

First, we applied the OFDM scheduling algorithm in the wireless system [15] to the fiber system. The main difference between the wireless and fiber systems is that the wireless channel is multipath. So the wireless channel gain is frequency selective, which means that the channel gain is related to both the user distance and the subcarrier frequency. While the fiber channel is quite fixed, once the ONU distance is set, the channel gain is almost set. Thus we modified the wireless OFDM three-dimensional DBA algorithm and applied it for upstream access in OFDMA-PONs. The proposed Algorithm 1 consists of the following two main procedures.

Algorithm 1 Subcarrier allocation

```
Step 1 (Initialization):
  For $k \leftarrow 1$ to $K$
      $S_k = S_{\min} = R_k/D_{\max}$;
      $\Delta P_k = [\mathrm{f}(1) - f(0)]/a_k^2$;
  /*Every user is assigned with max modulation format, allocated with min number of subcarriers, leading to the surplus of system subcarriers.*/
  /* Subcarrier assignment iteration. The average transmission power variation $\Delta P_k$, will be updated in each iteration.*/
Step 2 (Subcarrier assignment iteration):
  While $\sum_{k=1}^{K} S_k < N$
      $k = \arg\max_{k' \in \{1,2,\ldots,K\}} \Delta P_{k'}$;
      $S_k = S_k + 1$;
      $\Delta P_k = [S_k * f(R_k/S_k) - (S_k + 1) * f(R_k/S_k + 1)]/a_k^2$;
```

Algorithm 1 Bit allocation

```
For $k \leftarrow 1$ to $K$
  Step 1 (Initialization):
    For each $n \in L_k$
      $c_{k,n} = 1$,
      $\Delta P_n = [f(1) - f(0)]/a_k^2$;
    For each $n \in \{1, 2, \ldots, N\} \backslash L_k$
      $c_{k,n} = 0$;
  Step 2:
    While $\sum_{n=1}^{N} c_{k,n} < R_k$
      $n = \arg\max_{n' \in L_k} \Delta P_{n'}$;
      $c_{k,n} = c_{k,n} + 1$;
      $\Delta P_n = [f(c_{k,n} + 1) - f(c_{k,n})]/a_k^2$.
```

The first procedure assigns N subcarriers to K ONUs with the maximal modulation format. Since we did not know the exact number of bits/symbol each subcarrier should be assigned, we just assumed that each subcarrier carries the maximal number of bits/symbol in each ONU and calculated the transmitting power of each ONU, i.e., $S_k * f(R_k/S_k)$, roughly.

Although the data rate requirement of each ONU can be met in the first procedure, the total transmission power of ONUs P_T is not optimal. Therefore, the second procedure based on the greedy algorithm [18,19] was used in each ONU to further reduce the transmitting power under the condition that the data rate requirement of each ONU is met.

C. Second Proposed Algorithm

According to the analysis, we can know that Algorithm 1 separates the problem into two procedures: subcarrier allocation and bit allocation. However, the two procedures are not independent of each other. Subcarrier allocation and bit allocation will influence each other. Therefore, Algorithm 1 is a suboptimal algorithm but not optimal for solving the mathematical model to minimize the total transmission power. In order to find a better algorithm to solve this model, i.e., finding the best assignment of $c_{k,n}$ within each time slot and keeping the total transmitting power minimized, we further proposed Algorithm 2, where we allocated the subcarrier and bit in the same procedure and always kept the real transmitting power as the optimization function. The main procedure is as follows.

Algorithm 2 Subcarrier and bit allocation

```
Step 1 (Initialization):
  For $k \leftarrow 1$ to $K$
    $S_k = S_{\max} = R_k/D_{\min}$;
    For each $n \in L_k$,
      $c_{k,n} = D_{\min}$;
      $Update(c_{k,n}, \Delta P_k)$.
    For each $n \notin \{L_1 \cup L_2 \cup \ldots \cup L_K\} \backslash L_k$,
      $c_{k,n} = 0$.
/*Every user is assigned with min modulation format, allocated with max number of subcarriers, leading to the shortage of system subcarriers.*/
Step 2 (Subcarrier, bit assignment iteration):
  While $\sum_{k=1}^{K} S_k > N$
    $k = \arg\min_{k' \in \{1,2,\ldots,K\}} \Delta P_{k'}$;
    $Update(c_{k,n}, \Delta P_k)$;
    $S_k = S_k - 1$;
    Delete one subcarrier in $L_k$.
```

Algorithm 2 $Update(c_{k,n}, \Delta P_k)$

```
Step 1:
  For the $k$th ONU, find OFDM subcarrier $N_1$ with minimum number of bits/symbol assigned.
Step 2:
  While $c_{k,N1} > 0$
    For the $k$th ONU, find OFDM subcarrier $N_2$ with second minimum number of bits/symbol assigned;
    Update $\Delta P_k$ by Eq. (4);
      $$\Delta P_k = \{[f(c_{k,N_2} + 1) - f(c_{k,N_2})] - [f(c_{k,N_1} + 1) - f(c_{k,N_1} - 1)]\}/a_k^2 \quad (4)$$
    Assign one bit in subcarrier $N_1$ to subcarrier $N_2$; thus $c_{k,N1} = c_{k,N1} - 1$ and $c_{k,N2} = c_{k,N2} + 1$.
```

In the proposed Algorithm 2, first, we initialized the subcarrier with the maximal number so that the transmitting power is the minimum. However, in general, the total number of subcarriers N is less than the maximal requirement of $\sum_{k=1}^{K} S_k$ in initialization. Hence, we should reduce the subcarrier number assigned to each ONU. Second, in order to solve the problem of the shortage of subcarriers, we found the kth ONU, in which the power variation ΔP_k is the smallest of all. Then we reduced one subcarrier in the kth ONU and enhanced its transmitting power by ΔP_k. This process was realized in the subalgorithm of $Update(c_{k,n}, \Delta P_k)$. In this subalgorithm, we first figured out how the data rate is reassigned among the subcarriers in the kth ONU. Then we calculated ΔP_k and reassigned the bit allocation in the subcarrier. Finally, we recursively found out the ONU with minimum power variation ΔP, deleted one subcarrier assigned to this ONU, and enhanced its transmitting power by ΔP until the system subcarriers were enough, namely, $\sum_{k=1}^{K} S_k \le N$.

After running Algorithm 1 or Algorithm 2, we got the best assignment of $c_{k,n}$ within each time slot. The traffic requirement of each ONU is met, and the minimized total transmitting power P_T can be obtained from Eq. (1).

IV. COMPLEXITY ANALYSIS AND SIMULATION RESULTS

In this section, we first analyze the time complexity of the three-dimensional DBA algorithm and then simulate the signal transmitting power of ONUs by the proposed algorithms. For comparison, the traditional two-dimensional DBA algorithm is considered under the same conditions.

A. Complexity Analysis

We considered the running time complexity of the proposed algorithms and the two-dimensional DBA algorithm.

Theorems 1, 2, and 3 are the time complexity of the algorithms. The proofs of these theorems are in Appendix A.

Theorem 1: The time complexity of the traditional two-dimensional algorithm is $O(KN)$.

Theorem 2: The worst-case time complexity of the proposed Algorithm 1 is $O(KN^2)$.

Theorem 3: The worst-case time complexity of the proposed Algorithm 2 is $O(KN^2)$.

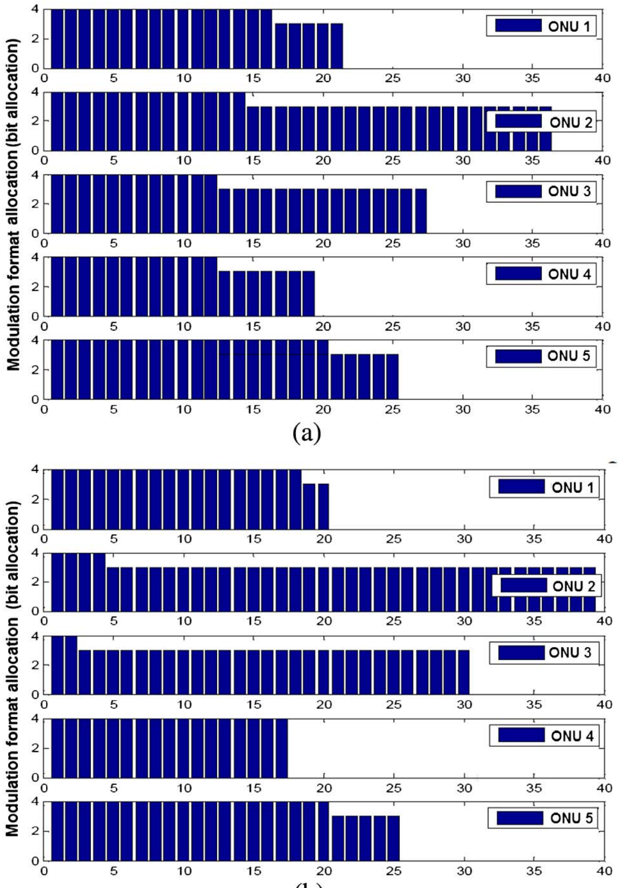
Due to the difference of the scheduling dimension, it is no wonder that the complexity of the two-dimensional algorithm is simpler than that of the two proposed algorithms. Fortunately, due to the high speed and low power consumption of the digital signal processing (DSP) hardware [20] and the small number of N, K, and $D_{\max}$, the extra electrical power consumption due to complex bit allocation in the three-dimensional DBA algorithm is negligible when we compare the power consumption of the two-dimensional algorithm with the two proposed three-dimensional algorithms. The running time of Algorithm 1 and Algorithm 2 is upper bounded by KN^2, which is a polynomial expression. Therefore, the time complexity of the proposed two algorithms is polynomial.

B. Simulation Results

In the simulation experiment, we assumed that the distances between ONUs and the OLT are randomly distributed in the interval from 0 to 20 km and the packet interarrival time obeys the Pareto distribution in the network. First, we established a small system including 5 ONUs, 128 OFDM subcarriers, and 1.28 GHz bandwidth. Figure 3 indicates how many subcarriers of each ONU are assigned and the number of bits per OFDM symbol assigned to each subcarrier in Algorithm 1 and Algorithm 2 for the total bandwidth demand of 4.5 Gb/s. Then we established another bigger system with 32 ONUs, 1024 subcarriers, and total system bandwidth of 10.24 GHz. Figure 4 shows the total transmitting power of ONUs versus the total data demand of all ONUs, with the five-ONU system in Fig. 4(a) and the 32-ONU system in Fig. 4(b). The contrastive algorithm is the traditional two-dimensional algorithm with fixed 16-QAM modulation format for each subcarrier, which is the minimal modulation format to meet the maximum bandwidth demand in Figs. 4(a) and 4(b).

During the simulation process, the bandwidth demand for each ONU is randomly assigned and set $D = \{1, 2, \ldots, 10\}$, namely, $D_{\min} = 1$ and $D_{\max} = 10$, respectively. As we mentioned above, the electrical power contributes a large part of the ONU total transmitting power and the optical power of ONUs is similar to each other. We ignored the optical power of ONUs and calculated the relative value of the electrical transmitting power of all ONUs in the simulation.

From Figs. 3(a) and 3(b), we can get that the allocation results of the subcarrier and bit in the proposed Algorithm 2 are better than that in the proposed Algorithm 1. Compared with Algorithm 1, Algorithm 2 has fewer subcarriers with high modulation format, which means the overall transmitting power of ONUs by Algorithm 2 is smaller than that by Algorithm 1. The reason is shown below. We know that Algorithm 1 divides the problem into two procedures: subcarrier allocation and bit allocation. Actually, as we mentioned in the introduction, the two procedures are related to each other. Thus, Algorithm 1 is not optimal but a suboptimal algorithm. However, considering the relationship between subcarrier allocation and bit allocation, Algorithm 2 completes subcarrier allocation and bit allocation in the same procedure. Therefore, Algorithm 2 can find the best assignment of $c_{k,n}$ within each time slot and keep the signal transmitting power of ONUs minimized.



(a)

(b)

Fig. 3. For 4.5 Gb/s total bandwidth demand with five ONUs, 128 OFDM subcarriers, and 1.28 GHz bandwidth, subcarrier, and bit allocation results for (a) Algorithm 1 and (b) Algorithm 2.

From Figs. 4(a) and 4(b), we can know that compared with the traditional two-dimensional DBA algorithm, the transmitting power of ONUs is reduced by ~30% and ~50% using the proposed Algorithm 1 and Algorithm 2, respectively. It implies that the three-dimensional scheduling algorithm outperforms the two-dimensional scheduling algorithm in minimizing the total transmitting power when the traffic requirement of each ONU is met. Furthermore, the performance of the proposed Algorithm 2 is better than that of Algorithm 1 in solving the mathematical model to find the best assignment of $c_{k,n}$ within each time slot and minimize the total transmitting power of ONUs.

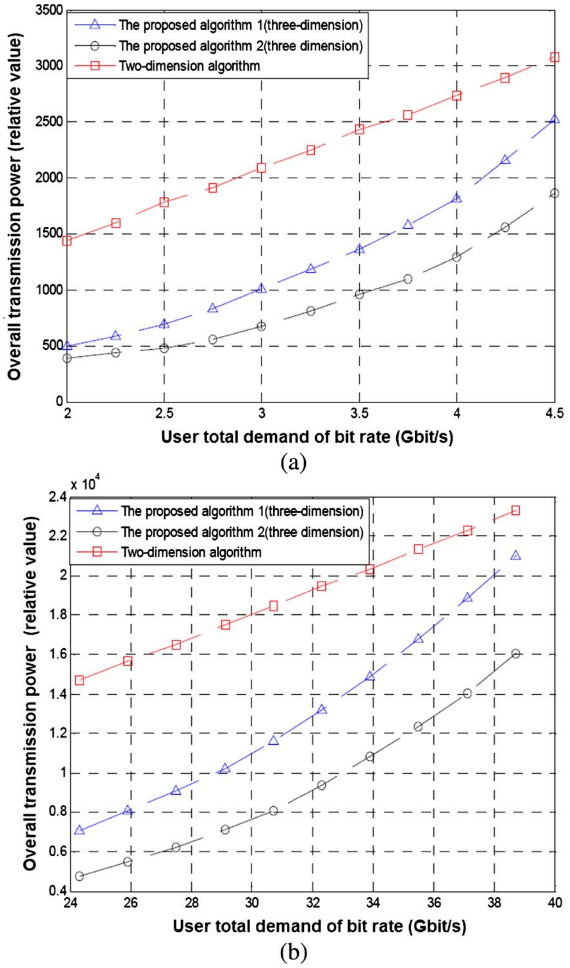


Fig. 4. Relative transmission power. (a) Overall transmitting power of ONUs versus user's data rate for different algorithms with five ONUs, 128 OFDM subcarriers, and 1.28 GHz bandwidth. (b) Overall transmitting power of ONUs versus user's data rate for different algorithms with 32 ONUs, 1024 OFDM subcarriers, and 10.24 GHz bandwidth.

As we mentioned before, although the time complexity of the proposed algorithms is higher than that of the two-dimensional algorithm, it is reasonable not considering the extra electrical power consumption in three-dimensional algorithms due to the high speed of DSP hardware. The comparison of the power consumption between two-dimensional DBA and the two proposed algorithms of three-dimensional DBA is relatively fair.

In the simulation experiment, for simplicity the algorithms are based on a fixed time slot. However, the time slot can be changed in each frame cycle and the proposed algorithms can handle it well. When the time slot is variable, the flexibility in bandwidth allocation will not suffer.

V. CONCLUSION

We have proposed two three-dimensional DBA algorithms in OFDMA-PONs—dynamically allocating the time slot, subcarrier, and modulation format of the ONUs—to minimize the total transmitting power of ONUs. From simulation results, we know that both of the proposed algorithms are better than the traditional two-dimensional algorithm in terms of minimizing the total transmitting power of ONUs. Hence, it implies that a subcarrier with an adaptive modulation scheme rather than a fixed one can achieve much lower transmitting power. Moreover, the performance of the proposed Algorithm 2 is better than that of the proposed Algorithm 1 by allocating the subcarrier and bit of each time slot in a single procedure rather than in two procedures. The complexity of the two proposed algorithms is proved to be polynomial time.

APPENDIX A

The time complexity of the two-dimensional DBA algorithm and the proposed three-dimensional DBA algorithms is analyzed in this appendix.

The typical two-dimensional DBA algorithm [9] can be shown as follows:

```
For $n \leftarrow 1$ to $N$
    $k = \arg \max_{k' \in \{1,2,\ldots,K\}} R_{k'}$
        $R_k = R_k - D_{\text{fix}}$ //$D_{\text{fix}}$ is the fixed bits each OFDM subcarrier can transmit.
        $L_k = L_k \cup \{n\}$
        do if $R_k <= 0$
            then $R_k = 0$
```

Proof of Theorem 1:

$$T = O(N)[O(K) + O(1)] = O(NK) + O(N) = O(NK) \qquad \square$$

Recall that $f(n) = O(g(n))$ means that there exist a constant c and integer n_0 such that $0 \leq f(n) \leq cg(n)$ for all $n > n_0$.

Proof of Theorem 2:

$$\begin{aligned}
T &= \underbrace{O(K)+O(N)[O(K)+O(1)]}_{\text{Subcarrier allocation}} \\
&\quad + \underbrace{O(K)\{O(S_k)+O(N-S_k)+O(R_k)[O(S_k)+O(1)]\}}_{\text{Bit allocation}} \\
&= \underbrace{O(K)+O(NK)}_{\text{Subcarrier allocation}} + \underbrace{O(K)[O(S_k)+O(N-S_k)+O(R_kS_k)]}_{\text{Bit allocation}} \\
&= \underbrace{O(K)+O(NK)}_{\text{Subcarrier allocation}} \\
&\quad + \underbrace{O(K)[O(S_k)+O(N-S_k)+O(D_{\max}S_k^2)](\text{for} R_k = S_k D_{\max})}_{\text{Bit allocation}} \\
&= \underbrace{O(K)+O(NK)}_{\text{Subcarrier allocation}} \\
&\quad + \underbrace{O(K)[O(N)+O(N)+O(D_{\max}N^2)]}_{\text{Bit allocation}} \ (\text{for } S_k \leq N) \\
&= \underbrace{O(NK)}_{\text{Subcarrier allocation}} + \underbrace{O(D_{\max}KN^2)}_{\text{Bit allocation}} \\
&= O(D_{\max}KN^2) \\
&= O(KN^2) \ \ (\text{for } D_{\max} \text{ is a constant}) \qquad \square
\end{aligned}$$

Proof of Theorem 3:

We first prove that the time complexity of the subalgorithm $Update(c_{k,n}, \Delta P_k)$, which is noted as T_{sub}, is $O(S_k)$:

$$\begin{aligned} T_{\text{sub}} &= O(S_k) + O(D_{\max})[O(S_k) + O(1)] \\ &= O(S_k) + O(D_{\max}S_k) = O(S_k) \\ &\text{(for } D_{\max} \text{ is a constant).} \end{aligned}$$

Then the time complexity of Algorithm 2 is proved as follows:

$$\begin{aligned} T &= \underbrace{O(K)\left\{O(1) + O(S_k)[O(1) + T_{\text{sub}}] + O\left(\sum S_k\right)O(1)\right\}}_{\text{Algorithm 2–Step 1}} \\ &\quad + \underbrace{O\left(\sum S_k - N\right)[O(K) + T_{\text{sub}} + O(1)]}_{\text{Algorithm 2–Step 2}} \\ &= \underbrace{O(K)\left[O(S_k^2) + O\left(\sum S_k\right)\right]}_{\text{Algorithm 2–Step 1}} \\ &\quad + \underbrace{O\left(\sum S_k - N\right)[O(K) + O(S_k)]}_{\text{Algorithm 2–Step 2}} \\ &= \underbrace{O(K)O(N^2)}_{\text{Algorithm 2–Step 1}} + \underbrace{O(N)[O(K) + O(N)]}_{\text{Algorithm 2–Step 2}} \quad (\text{for } S_k = O(N)) \\ &= \underbrace{O(KN^2)}_{\text{Algorithm 2–Step 1}} + \underbrace{O(NK) + O(N^2)}_{\text{Algorithm 2–Step 2}} = O(KN^2) \qquad \square \end{aligned}$$

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